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A New Characterization of Uniform Motion under a Variable Force in Relativistic Mechanics.

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***Abstract* -- This paper presents a new Characterization of Uniform Motion under a variable force in Relativistic Mechanics. Here, we consider the problems of Logical aspects in mechanics and cosmology, in which a variable force develops the whole systems of Mechanics. In the light of the variable force, here, it is proved in this paper, that the laws of motion are affected in Relativistic Mechanics in respect of invariance of force as well as invariance of acceleration.**

***Keywords*--Uniform Motion; Variable force, Relativistic Mechanics, Cosmology, Force and Acceleration.**

I. Introduction

Eddington (1) and Roser (4) are the pioneer workers of the present area. In fact, the present work is the extension of work done by Whittaker(5), Kumar et al. (2) and Kumar et al. (3). In this paper, we have studied analytically on Motion Under a Constant Force in Relativistic Mechanisms.

II. Mathematical Treatment Of The Problem

The concept of force plays a vital role to develop the whole systems of dynamics & Physics. In the light of the concept of force, Newton was able to synthesized two important components of the scientific revolution, the mechanical philosophy & mathematization of nature. With the help of laws of motion of known dynamics, it has been noticed that –

(a) Everybody continues in its rest – state or in state of motion in a straight line unless it is compelled to change that state by force impressed on it.
(b) The change of motion is proportional to the motive force impressed and is made in the direction of the straight lines in which that force is impressed.
(c) To every action there is always opposed an equal reaction or the mutual actions of two bodies upon each other are always equal.

The mathematical form of Newton's second law is

$$F = ma \qquad (1.1)$$

Where, F is the external force, m is the mass of the body & a is the acceleration of the body.

The mathematical form of the second law of motion established result (1.1) as explained the second law of motion as the rate of change of velocity is directly proportional to the force acting on a body and inversely proportional to its mass. The main idea of 'Newtonian Mechanics' is that external agencies described by the force concept; are responsible for the acceleration and not the velocity or the time derivative of the acceleration. So, Newton's second law of motion is not a definition of force, although it is the only possible way of determining a force in many specific cases. As we know that the force is a quantitative vector measure of the interaction intensity. A procedure for measuring forces is established independently of the measurement of acceleration. Moreover, the second law of motion expressed the acceleration of a body subjected to the action of a force, followed by

$$F = m\frac{dv}{dt} \qquad (1.2)$$

where, $\frac{dv}{dt}$ is the acceleration, a.

That is, $$a = \frac{dv}{dt} \qquad (1.3)$$

In the absence of a force, that is, when

$$F = 0 \qquad (1.4)$$

$$V = \text{Constant} \qquad (1.5)$$

Joining (1.4) & (1.5) ; we are at the position to state that if no force acts on a body or if the resultant of the forces applied to a body is zero, the body will be either at rest or moving uniformly in a straight line. Hence, we find that the first law is not independent and is just corollary of the second law. The physical meaning of Newton's second law of motion is that the external conditions are defined by acceleration and not by the velocity or the time derivative of the acceleration. In classical mechanics, the external conditions are described using the concept of force. Newtonian mechanics retains the concepts of natural and forced motion, but only the uniform motion in a straight line (the first law) is accepted as natural motion.

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Aristotle's law of motion states that the velocity is proportional to force; whereas, Newton's law of motion (second) states that the acceleration is proportional to force. So, we find that external conditions determine the acceleration. These conditions are not responsible to determine the velocity of an object. If the second law is just a definition of force, then described force is in such a way that the motion is defined in accordance with Aristotle's law or in such a way that the force is proportional to the third time derivative of coordinates. The fact that this is not possible confirms that Newton's second law of motion cannot be treated as definition of force. Equation (1.2) can be considered as a law rather than a definition of force only if there exists an independent force. The physical meaning of this law lies not in that the force has a specific expression, but in that the force defines the second time derivatives of co-ordinates:

$$\frac{dv}{dt} = \frac{d^2r}{dt^2} \qquad (1.6)$$

Thus, the invariance of acceleration relative to Galilean transformations leads to the invariance of force. The simplest dynamical problem in classical mechanics is the motion of a body under constant force. Suppose a force F acts on a body of mass m for a time t in one dimensional motion; the body is assumed to be initially at rest, and ends up with a speed V. Then –

$$Ft = mV = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (1.7)$$

Where, m_0 is the rest-mass of the body,

$$\therefore \qquad \sqrt{1-\frac{v^2}{c^2}} = \frac{m_0 v}{Ft}$$

On squaring,

$$1 - \frac{v^2}{c^2} = \left(\frac{m_0 v}{Ft}\right)^2$$

or

$$c^2 = v^2\left\{1 + \left(\frac{m_0 v}{Ft}\right)^2\right\} \qquad (1.8)$$

and

$$v(t) = \frac{c}{\left\{1+\left(\frac{m_0 c}{Ft}\right)^2\right\}^{\frac{1}{2}}} \qquad (1.9)$$

This is a rather complex – looking result.

We consider the case, when $Ft << m_0 c$;
we have,

$$\left(\frac{m_0 c}{Ft}\right)^2 >> 1 \qquad (1.10)$$

Therefore,

$$v(t) \approx \frac{c}{\left(\frac{m_0 c}{Ft}\right)} = \frac{F}{m_0} t \qquad (1.11)$$

We consider, $Ft >> m_0 c$; we find that

$$\left(\frac{m_0 c}{Ft}\right)^2 \rightarrow 0 \qquad (1.12)$$

In spite of (1.12), we find that

$$v(t) \approx c \qquad (1.13)$$

III. CONCLUSION

The case $Ft << m_0 c$ corresponds to ordinary Newtonian mechanics; whereas, the case $Ft >> m_0 c$ displays the new-familiar property of a limiting variable speed c for motion under variable force.

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